CHAPTER 3
Linear Equations and Problem Solving

Section 3.1 Solving Linear Equations

1. \( x = 9 \)
2. \( 8z - 2 = 0 \)
   \( 8z - 2 + 2 = 0 + 2 \)
   \( 8z = 2 \)
   \( \frac{8z}{8} = \frac{2}{8} \)
   \( z = \frac{1}{4} \)

3. \( x = -6 \)
4. Subtraction

5. Subtraction

6. Multiplication

7. Subtraction

8. Addition

11. Addition

13. \( 5x + 15 = 0 \)
   \( 5x + 15 - 15 = 0 - 15 \)
   \( 5x = -15 \)
   \( \frac{5x}{5} = \frac{-15}{5} \)
   \( x = -3 \)

15. \( 10x + 10 = 0 \)
   \( 10x + 10 - 10 = 0 - 10 \)
   \( 10x = -10 \)
   \( \frac{10x}{10} = \frac{-10}{10} \)
   \( x = -1 \)

17. \( 9g - 18 = 0 \)
   \( 9g - 18 + 18 = 0 + 18 \)
   \( 9g = 18 \)
   \( \frac{9g}{9} = \frac{18}{9} \)
   \( g = 2 \)

19. \( 4x - 24 = 0 \)
   \( 4x - 24 + 24 = 0 + 24 \)
   \( 4x = 24 \)
   \( \frac{4x}{4} = \frac{24}{4} \)
   \( x = 6 \)

21. \( 2x + 52 = 0 \)
   \( 2x + 52 - 52 = 0 - 52 \)
   \( 2x = -52 \)
   \( \frac{2x}{2} = \frac{-52}{2} \)
   \( x = -26 \)

23. \( 8z - 2 = 0 \)
   \( 8z - 2 + 2 = 0 + 2 \)
   \( 8z = 2 \)
   \( \frac{8z}{8} = \frac{2}{8} \)
   \( z = \frac{1}{4} \)

25. \( \frac{x}{4} + 7 = 0 \)
   \( \frac{x}{4} + 7 - 7 = 0 - 7 \)
   \( \frac{x}{4} = -7 \)
   \( 4 \cdot \frac{x}{4} = 4(-7) \)
   \( y = -28 \)

27. \( -x + 9 = 0 \)
   \( -x + 9 - 9 = 0 - 9 \)
   \( -x = -9 \)
   \( -x = -9 \)
   \( -1 = -1 \)
   \( x = 9 \)

29. \( -5x - 15 = 0 \)
   \( -5x - 15 + 15 = 0 + 15 \)
   \( -5x = 15 \)
   \( -5x = 15 \)
   \( -5 = -5 \)
   \( x = -3 \)

31. \( -3p + 1 = 0 \)
   \( -3p + 1 - 1 = 0 - 1 \)
   \( -3p = -1 \)
   \( -3p = -1 \)
   \( -3 = -3 \)
   \( p = \frac{1}{3} \)

33. \( 3y - 2 = 2y \)
   \( 3y - 3y - 2 = 2y - 3y \)
   \( -2 = -y \)
   \( (-1)(-2) = (-1)(-y) \)
   \( 2 = y \)
35. \[4 - 7x = 5x\]
\[4 - 7x + 7x = 5x + 7x\]
\[4 = 12x\]
\[\frac{4}{12} = \frac{12x}{12}\]
\[\frac{1}{3} = x\]

37. \[4 - 5t = 16 + t\]
\[4 - 6t = 16\]
\[4 - 4 - 6t = 16 - 4\]
\[-6t = 12\]
\[-6t = 12\]
\[-6 = -6\]
\[t = -2\]

39. \[-3t + 5 = -3t\]
\[-3t + 3t + 5 = 3t + 3t\]
\[5 = 0\] (False)
This equation has no solution.

41. \[4x - 2 = 3x + 1\]
\[4x - 3x - 2 = 3x - 3x + 1\]
\[x - 2 = 1\]
\[x - 2 + 2 = 1 + 2\]
\[x = 3\]

43. \[4x - 6 = 4x - 6\]
\[4x - 4x - 6 = 4x - 4x - 6\]
\[-6 = -6\] Identity
This equation has infinitely many solutions.

45. \[2x + 4 = -3(x - 2)\]
\[2x + 4 = -3x + 6\]
\[5x + 4 = 6\]
\[5x + 4 - 4 = 6 - 4\]
\[5x = 2\]
\[\frac{5x}{5} = \frac{2}{5}\]
\[x = \frac{2}{5}\]

47. \[5(3 - x) = x - 12\]
\[15 - 5x = x - 12\]
\[15 - 5x - x = x - x - 12\]
\[15 - 6x = -12\]
\[15 - 15 - 6x = -12 - 15\]
\[-6x = -27\]
\[-6x = -27\]
\[-6 = -6\]
\[x = \frac{9}{2}\]

49. \[2x = -3x\]
\[2x + 3x = -3x + 3x\]
\[5x = 0\]
\[\frac{5x}{5} = \frac{0}{5}\]
\[x = 0\]

51. **Verbal Model:**

**Labels:**
- Perimeter = 225 (cm)
- Length of a side = \(x\) (cm)

**Equation:**
\[
\frac{225}{3} = \frac{3x}{3}
\]
\[75 = x\]

The length of each side is 75 centimeters.

53. **Verbal Model:**

\[\frac{2}{\text{Length}} + \frac{2}{\text{Width}} = \text{Perimeter}\]

**Labels:**
- Width = \(w\) (inches)
- Length = \(2w\) (inches)
- Perimeter = 120 (inches)

**Equation:**
\[
2(2w) + 2w = 120
\]
\[4w + 2w = 120
\]
\[6w = 120
\]
\[w = 20 \text{ and } 2w = 40
\]

The width of the flag is 20 inches and the length is 40 inches.
55. **Verbal Model:** 

\[ \text{Total revenue} = \text{Revenue from main floor seats} + \text{Revenue from balcony seats} \]

**Labels:**
- Total revenue = 5200 (dollars)
- Price per main floor seat = 10 (dollars per seat)
- Number of main floor seats = 400 (seats)
- Price per balcony seat = 8 (dollars per seat)
- Number of balcony seats = \( x \) (seats)

**Equation:**

\[
5200 = 400(10) + 8x \\
5200 = 4000 + 8x \\
5200 - 4000 = 4000 - 4000 + 8x \\
1200 = 8x \\
1200 \div 8 = 8x \\
150 = x
\]

There were 150 balcony seats sold.

57. Yes. Subtract the cost of parts from the total to find the cost of labor. Then divide the labor cost by 44 to find the number of hours spent on labor.

**Verbal Model:** 

\[ \text{Cost of parts} + \text{Hourly cost of labor} \cdot \text{Hours of labor} = \text{Total cost} \]

**Labels:**
- Cost of parts = 285 (dollars)
- Hourly cost of labor = 44 (dollars per hour)
- Hours of labor = \( t \) (hours)
- Total cost = 384 (dollars)

**Equation:**

\[
285 + 44t = 384 \\
285 - 285 + 44t = 384 - 285 \\
44t = 99 \\
44t \div 44 = 99 \div 44 \\
t = 9 \div 4 \\
\frac{9}{4} \text{ hours or } 2 \frac{1}{4} \text{ hours. This answer could also be expressed as 2.25 hours or as 2 hours and 15 minutes.}
\]

59. **Verbal Model:** 

\[ \text{Income for year} = 24 \cdot \frac{\text{Amount of each paycheck}}{\text{Bonus}} \]

**Labels:**
- Income for year = 37,120 (dollars)
- Amount of each paycheck = \( x \) (dollars per paycheck)
- Bonus = 2800 (dollars)

**Equation:**

\[
37,120 = 24x + 2800 \\
34,320 = 24x \\
34,320 \div 24 = 24x \div 24 \\
1430 = x
\]

Each paycheck will be $1430.
61. Verbal Model:  
\[
\text{Total pay} = \text{Pay per week} \times \text{Number of weeks} + \text{Pay for training}
\]

**Labels:**  
- Total pay = 2635 (dollars)
- Pay per week = 320 (dollars per week)
- Number of weeks = \(x\) (weeks)
- Pay for training = 75 (dollars)

**Equation:**
\[
2635 = 320x + 75
\]

You interned for 8 weeks.

63. Substitute the solution into the original equation and simplify each side.
\[
3x + 2 = 11
\]
\[
3(3) + 2 = 11
\]
\[
9 + 2 = 11
\]
\[
11 = 11 \checkmark
\]

65. You are trying to isolate the variable term on the left-hand side of the equation. To do this, you must eliminate the +2 by subtracting 2: add 2.

67. \(10x = 50\)
\[
\frac{10x}{10} = \frac{50}{10}
\]
\[
x = 5
\]

69. \(\frac{x}{3} = 10\)
\[
\frac{3(x)}{3} = 3 \cdot 10
\]
\[
x = 30
\]

Note: \(\frac{3}{3} = 1\) \(\frac{x}{3} = \frac{x}{3} = \frac{3x}{3} = x\)

71. \(15x - 3 = 15 - 3x\)
\[
15x + 3x - 3 = 15 + 3x - 3x
\]
\[
18x - 3 = 15
\]
\[
18x - 3 + 3 = 15 + 3
\]
\[
18x = 18
\]
\[
\frac{18x}{18} = \frac{18}{18}
\]
\[
x = 1
\]

73. \(2x - 5 + 10x = 3\)
\[
12x - 5 = 3
\]
\[
12x - 5 + 5 = 3 + 5
\]
\[
12x = 8
\]
\[
\frac{12x}{12} = \frac{8}{12}
\]
\[
x = \frac{2}{3}
\]

75. \(5t - 4 + 3t = 8t - 4\)
\[
8t - 4 = 8t - 4 \text{ Identity}
\]

The equation has infinitely many solutions.

77. \(3(2 - 7x) = 3(4 - 7x)\)
\[
6 - 21x = 12 - 21x
\]
\[
6 - 21x + 21x = 12 - 21x + 21x
\]
\[
6 \neq 12
\]

The equation has no solution.

79. Verbal Model:  
**Perimeter** = 2 \cdot **Length** + 2 \cdot **Width**

**Labels:**  
- Perimeter = 260 (meters)
- Width = \(w\) (meters)
- Length = \(\ell\) = \(w + 30\) (meters)

**Equation:**
\[
260 = 2(w + 30) + 2w
\]
\[
260 = 2w + 2w + 60
\]
\[
260 = 4w + 60
\]
\[
200 = 4w
\]
\[
\frac{200}{4} = \frac{4w}{4}
\]
\[
w = 50
\]

The length is 80 meters and the width is 50 meters.
81. Verbal Model: \[
\text{Total pay per hour} = \text{Pay per hour} + 0.60 \cdot \text{Number of units}
\]
Labels:
- Total pay per hour = 15.50 (total dollars per hour)
- Pay per hour = 8.30 (dollars per hour)
- Number of units = \(x\) (units)

Equation:
\[
15.50 = 8.30 + 0.60x
\]
\[
7.20 = 0.60x
\]
\[
\frac{7.20}{0.60} = x
\]
\[
12 = x
\]
You must produce 12 units per hour to earn $15.50 per hour.

83. Verbal Model: \[
\text{Total wages} = \text{Rate for job 1} \cdot \text{Time at job 1} + \text{Rate for job 2} \cdot \text{Time at job 2}
\]
Labels:
- Total wages = 400 (dollars)
- Rate for job 1 = 8.75 (dollars per hour)
- Time at job 1 = 30 (hours)
- Rate for job 2 = 11 (dollars per hour)
- Time at job 2 = \(x\) (hours)

Equation:
\[
400 = 8.75(30) + 11x
\]
\[
400 = 262.50 + 11x
\]
\[
137.50 = 11x
\]
\[
\frac{137.50}{11} = x
\]
\[
12.5 = x
\]
You must work 12.5 hours, or 12 hours and 30 minutes, at the second job.

85. Verbal Model: \[
\text{Weight of red box} + 3 \cdot \text{Weight of blue box} = 9 \cdot \text{Weight of blue box}
\]
Labels:
- Weight of red box = \(R\) (ounces)
- Weight of blue box = 1 (ounce)

Equation:
\[
R + 3(1) = 9(1)
\]
\[
R + 3 = 9
\]
\[
R + 3 - 3 = 9 - 3
\]
\[
R = 6
\]
The red box weighs 6 ounces. If you remove three blue boxes from each side, the scale would still balance. The red box would balance the remaining six blue boxes, showing that the red box weighs 6 ounces. This illustrates the addition (or subtraction) property of equality.

87. False.
Multiplying each side of an equation by 0 yields 0 = 0. For example, multiplying each side of the equation \(3x = 9\) by 0, yields the equation 0 = 0. The equation \(3x = 9\) has one solution, \(x = 3\); however, the equation 0 = 0 is an identity and has infinitely many solutions. Because the original equation \(3x = 9\) and the resulting equation 0 = 0 have different solutions, they are not equivalent equations.

89. False.
An odd integer could be expressed as \(2m + 1\) and an even integer could be expressed as \(2n\). When they are added together, \((2m + 1) + 2n = 2m + 1 + 2n\) or \(2m + 2n + 1\). This expression could be written as \(2(m + n) + 1\); it represents an odd number because it is not a multiple of 2.
Section 3.2 Equations That Reduce to Linear Form

91. \(-3 < \cdots < \frac{2}{1} \leq \cdots \leq 4\)

93. \(-\frac{1}{2} \leq \cdots \leq 2.6 \leq 4\)

95. \(x - 8 = -9\)
   (a) \(x = -1\)
       
       \(-1 - 8 = -9\)
       
       \(-9 = -9\)
       
       \(-1\) is a solution.
   (b) \(x = 2\)
       
       \(2 - 8 = -9\)
       
       \(-6 \neq -9\)
       
       \(2\) is not a solution.

Section 3.2 Equations That Reduce to Linear Form

1. \(2(y - 4) = 0\)
   \[2y - 8 = 0\]
   \[2y - 8 + 8 = 0 + 8\]
   \[2y = 8\]
   \[y = 4\]

3. \(12(x - 3) = 0\)
   \[12x - 36 = 0\]
   \[12x - 36 + 36 = 0 + 36\]
   \[12x = 36\]
   \[x = 3\]

5. \(7(x + 5) = 49\)
   \[7x + 35 = 49\]
   \[7x + 35 - 35 = 49 - 35\]
   \[7x = 14\]
   \[x = 2\]

7. \(-5(t + 3) = 10\)
   \[5t + 15 = 10\]
   \[5t + 15 - 15 = 10 - 15\]
   \[5t = -5\]
   \[t = -5\]

9. \(15(x + 1) - 8x = 29\)
   \[15x + 15 - 8x = 29\]
   \[7x + 15 = 29\]
   \[7x + 15 - 15 = 29 - 15\]
   \[7x = 14\]
   \[x = 2\]

11. \(4 - (z + 6) = 8\)
   \[4 - z - 6 = 8\]
   \[-z - 2 = 8\]
   \[-z - 2 + 2 = 8 + 2\]
   \[-z = 10\]
   \[\frac{-z}{-1} = \frac{10}{-1}\]
   \[z = -10\]